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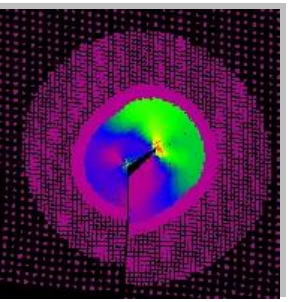

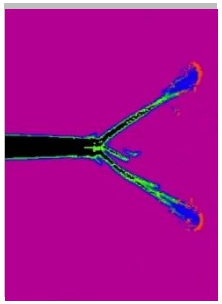

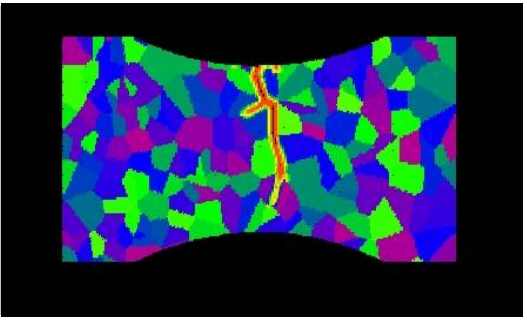

Peridynamic model for fatigue cracks

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Symposium on Computational Fracture Mechanics
Purdue University
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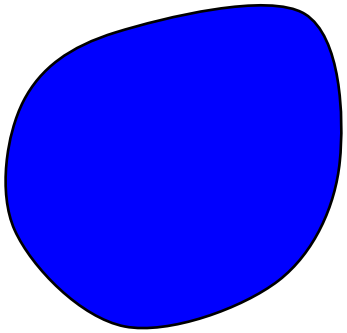
Outline

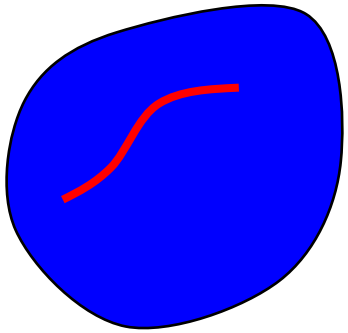
- Peridynamics background
- Cyclic bond strain
- Fatigue model
  - Nucleation phase
  - Growth phase
  - Heterogeneity

Purpose of peridynamics*

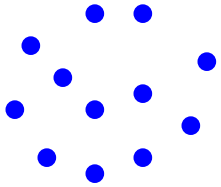
- To unify the mechanics of continuous and discontinuous media within a single, consistent set of equations.



Continuous body



Continuous body with a defect



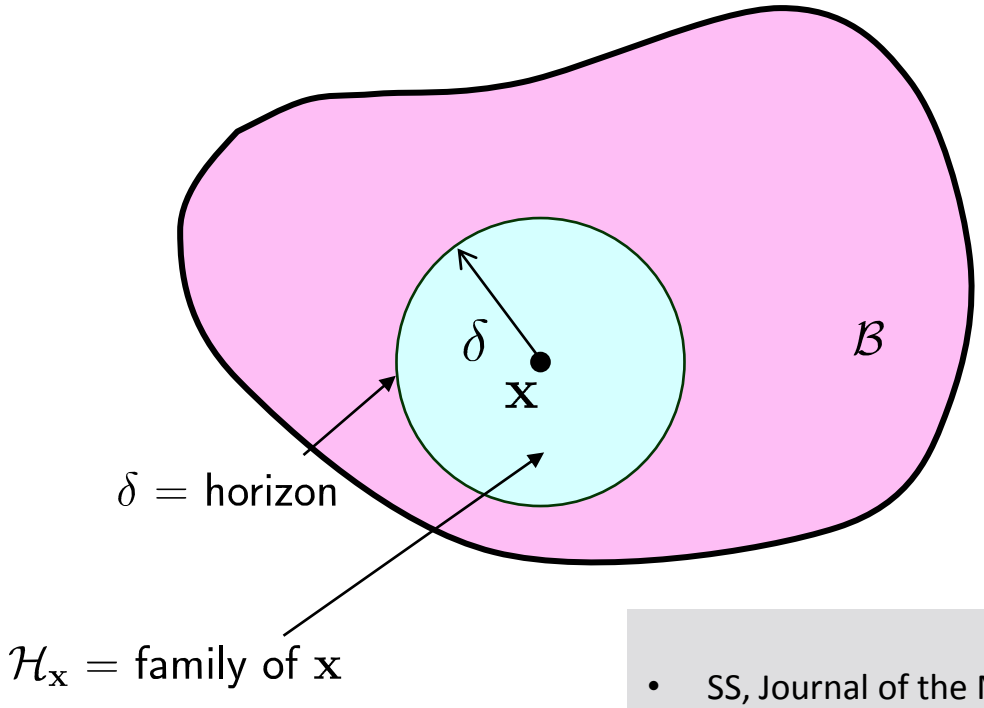
Discrete particles

- Why do this?
  - Avoid coupling dissimilar mathematical systems (A to C).
  - Model complex fracture patterns.
  - Communicate across length scales.

* Peri (near) + dyn (force)

Peridynamics basics: Horizon and family

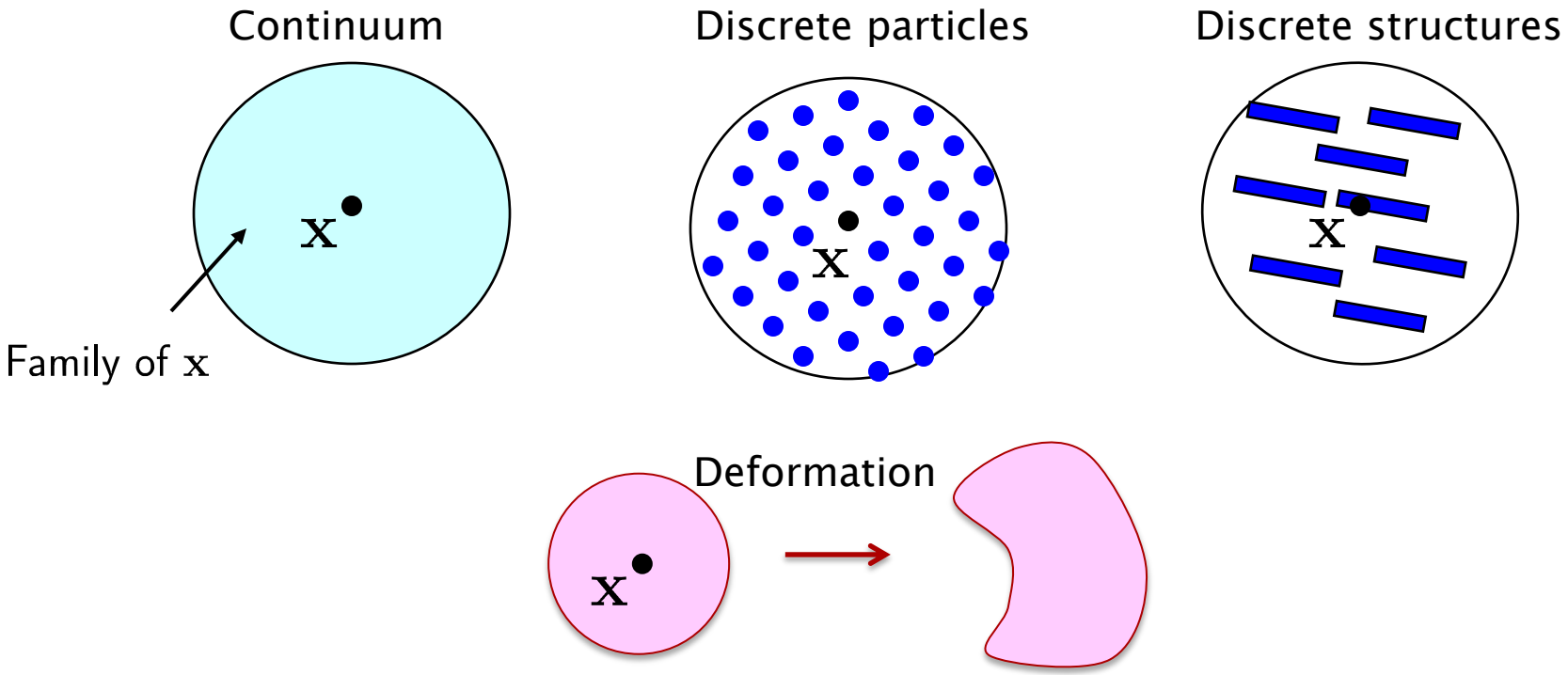
- Any point $\mathbf{x}$ interacts directly with other points within a distance δ called the "horizon."
- The material within a distance δ of $\mathbf{x}$ is called the "family" of $\mathbf{x}$, $\mathcal{H}_{\mathbf{x}}$.



General references

- SS, Journal of the Mechanics and Physics of Solids (2000)
- SS and R. Lehoucq, Advances in Applied Mechanics (2010)
- Madenci & Oterkus , *Peridynamic Theory & Its Applications* (2014)

Peridynamic nonlocality: Strain energy at a point



- Key assumption: the strain energy density at $W(\mathbf{x})$ is determined by the deformation of its family.

Potential energy minimization yields the peridynamic equilibrium equation

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- Potential energy:

$$\Phi = \int_{\mathcal{B}} (W - \mathbf{b} \cdot \mathbf{y}) \, dV_{\mathbf{x}}$$

where W is the strain energy density, $\mathbf{y}$ is the deformation map, $\mathbf{b}$ is the applied external force density, and $\mathcal{B}$ is the body.

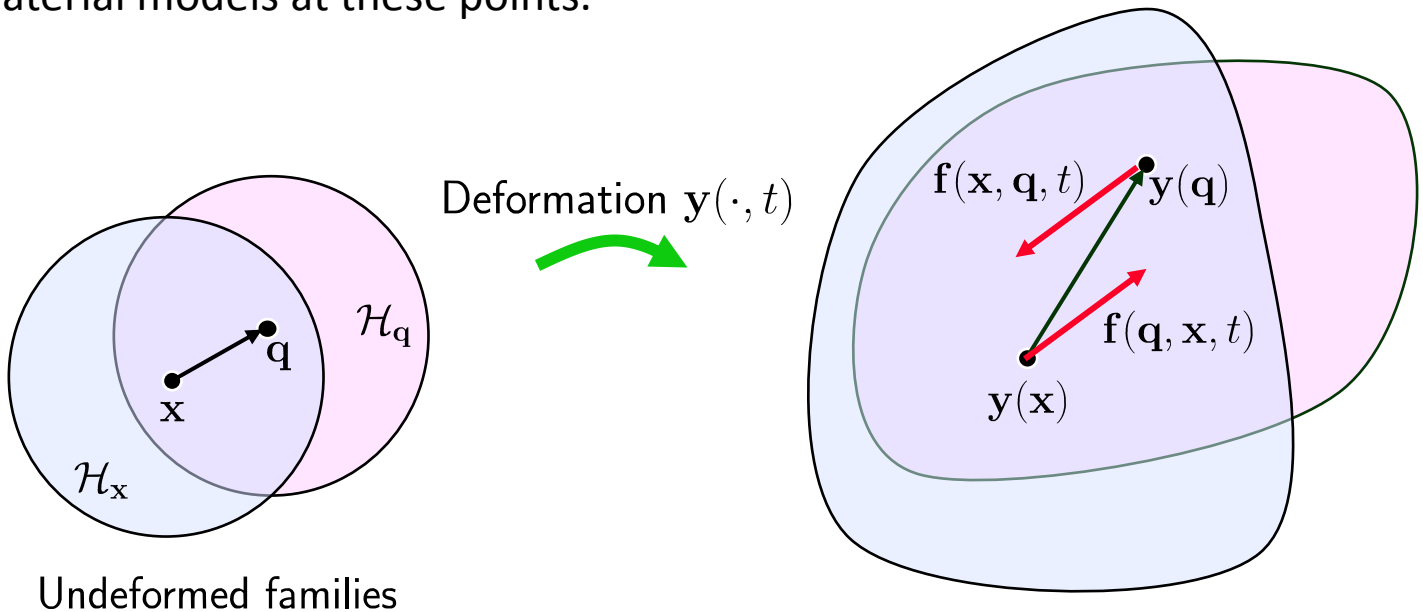
- Euler-Lagrange equation is the equilibrium equation:

$$\int_{\mathcal{H}_{\mathbf{x}}} \mathbf{f}(\mathbf{q}, \mathbf{x}) \, dV_{\mathbf{q}} + \mathbf{b}(\mathbf{x}) = 0$$

for all $\mathbf{x}$. $\mathbf{f}$ is the *pairwise bond force density*.

Material model determines bond forces

- Bond forces depend on the deformation of the families of both $\mathbf{x}$ and $\mathbf{q}$, together with the material models at these points.



Undeformed families

Deformed families and bond forces

In state notation: $\mathbf{f}(\mathbf{q}.\mathbf{x}) = \mathbf{T}[\mathbf{x}]\langle \mathbf{q} - \mathbf{x} \rangle - \mathbf{T}[\mathbf{q}]\langle \mathbf{x} - \mathbf{q} \rangle$

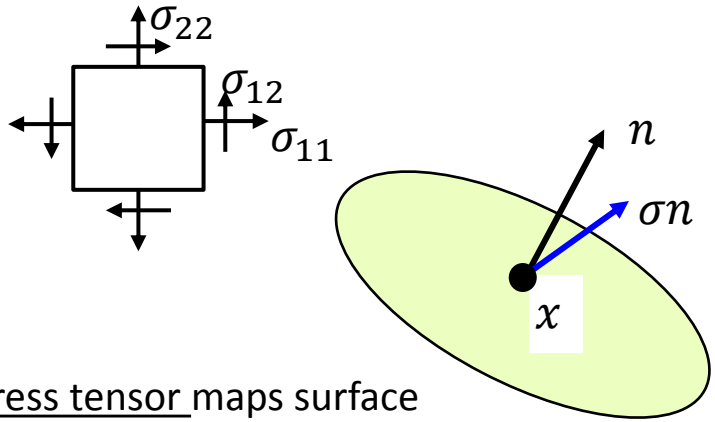
From mat model at $\mathbf{x}$

From mat model at $\mathbf{q}$

The nature of internal forces

Standard theory

Stress tensor field
(assumes continuity of forces)



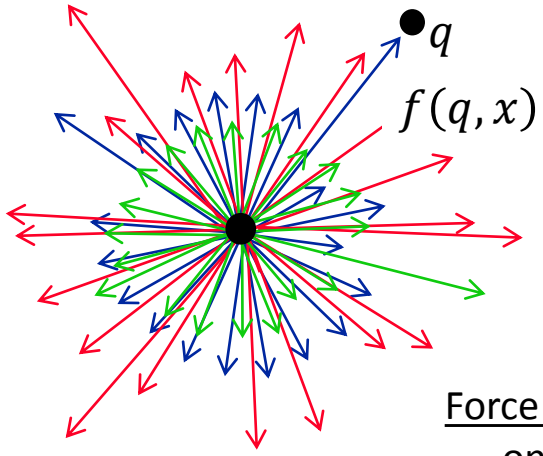
Stress tensor maps surface normal vectors onto surface forces

$$\rho\ddot{u}(x,t) = \nabla \cdot \sigma(x,t) + b(x,t)$$

Differentiation of surface forces

Peridynamics

Bond forces between neighboring points
(allowing discontinuity)



Force state maps bonds onto bond forces

$$\rho\ddot{u}(x,t) = \int_{H_x} f(q,x)dV_q + b(x,t)$$

Summation over bond forces

Peridynamic vs. local equations

- The structures of the theories are similar, but peridynamics uses nonlocal operators.

Relation	*Peridynamic theory*	*Standard theory*
Kinematics	$\underline{\mathbf{Y}}\langle \mathbf{q}-\mathbf{x}\rangle = \mathbf{y}(\mathbf{q})-\mathbf{y}(\mathbf{x})$	$\mathbf{F}(\mathbf{x}) = \dfrac{\partial \mathbf{y}}{\partial \mathbf{x}}(\mathbf{x})$
Linear momentum balance	$\rho\ddot{\mathbf{y}}(\mathbf{x}) = \displaystyle\int_{\mathcal{H}} \Big(\mathbf{t}(\mathbf{q},\mathbf{x}) - \mathbf{t}(\mathbf{x},\mathbf{q})\Big)\, dV_{\mathbf{q}} + \mathbf{b}(\mathbf{x})$	$\rho\ddot{\mathbf{y}}(\mathbf{x}) = \nabla\cdot\boldsymbol{\sigma}(\mathbf{x}) + \mathbf{b}(\mathbf{x})$
Constitutive model	$\mathbf{t}(\mathbf{q},\mathbf{x}) = \underline{\mathbf{T}}\langle \mathbf{q}-\mathbf{x}\rangle, \qquad \underline{\mathbf{T}} = \hat{\underline{\mathbf{T}}}(\underline{\mathbf{Y}})$	$\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}(\mathbf{F})$
Angular momentum balance	$\displaystyle\int_{\mathcal{H}} \underline{\mathbf{Y}}\langle \mathbf{q}-\mathbf{x}\rangle \times \underline{\mathbf{T}}\langle \mathbf{q}-\mathbf{x}\rangle\, dV_{\mathbf{q}} = \mathbf{0}$	$\boldsymbol{\sigma} = \boldsymbol{\sigma}^T$
Elasticity	$\underline{\mathbf{T}} = W_{\underline{\mathbf{Y}}}$ (Fréchet derivative)	$\boldsymbol{\sigma} = W_{\mathbf{F}}$ (tensor gradient)
First law	$\dot{\varepsilon} = \underline{\mathbf{T}} \bullet \underline{\dot{\mathbf{Y}}} + q + r$	$\dot{\varepsilon} = \boldsymbol{\sigma}\cdot\dot{\mathbf{F}} + q + r$

$$\underline{\mathbf{T}} \bullet \underline{\dot{\mathbf{Y}}} := \int_{\mathcal{H}} \underline{\mathbf{T}}\langle \boldsymbol{\xi}\rangle \cdot \underline{\dot{\mathbf{Y}}}\langle \boldsymbol{\xi}\rangle\, dV_{\boldsymbol{\xi}}$$

EMU numerical method

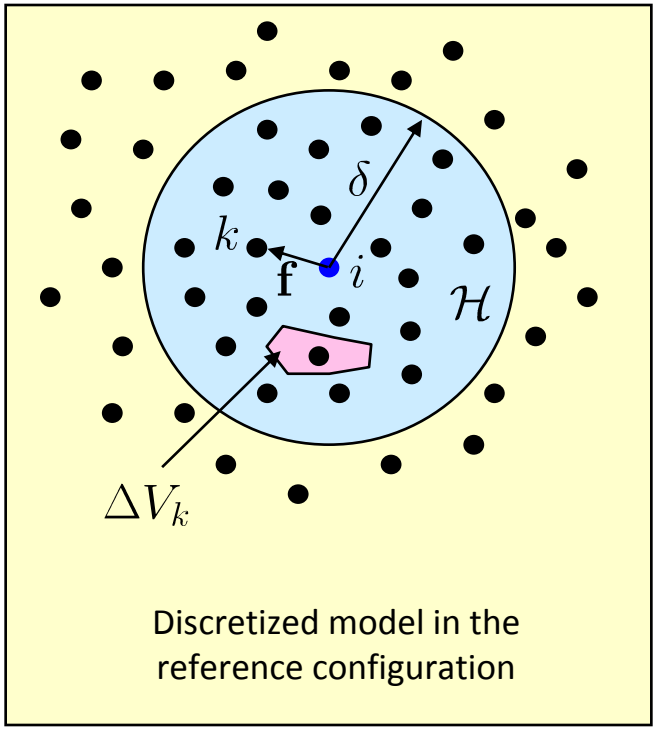
- Integral is replaced by a finite sum: resulting method is meshless and Lagrangian.

$$\rho\ddot{\mathbf{y}}(\mathbf{x},t) = \int_{\mathcal{H}} \mathbf{f}(\mathbf{q},\mathbf{x},t)\, dV_{\mathbf{q}} + \mathbf{b}(\mathbf{x},t)$$

$$\downarrow$$

$$\rho\ddot{\mathbf{y}}_i^n = \sum_{k\in\mathcal{H}} \mathbf{f}(\mathbf{x}_k,\mathbf{x}_i,t)\, \Delta V_k + \mathbf{b}_i^n$$

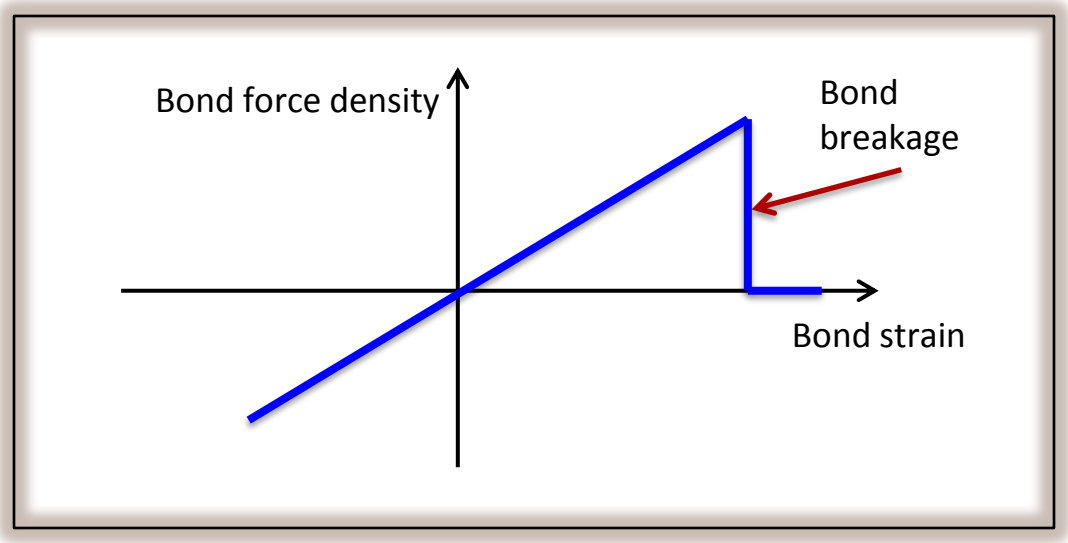
  - Looks a lot like MD.



Discretized model in the reference configuration

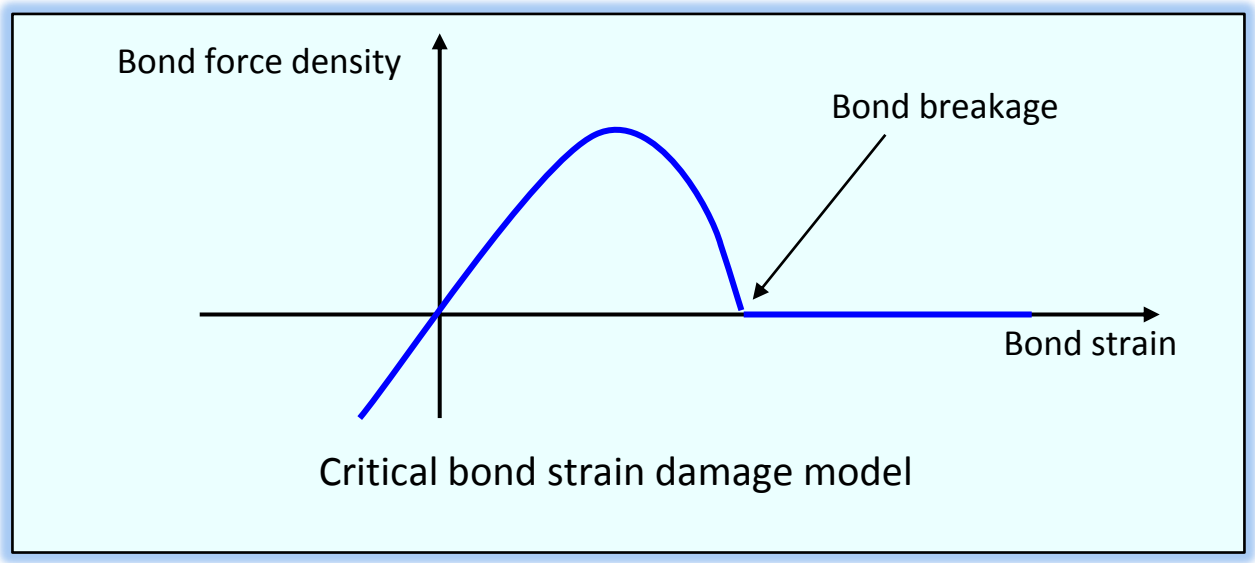
Bond based material models

- If each bond response is independent of the others, the resulting material model is called bond-based.
- The material model is then simply a graph of bond force density vs. bond strain.
- Damage can be modeled through bond breakage.
- Bond response is calibrated to:
  - Bulk elastic properties.
  - Critical energy release rate.



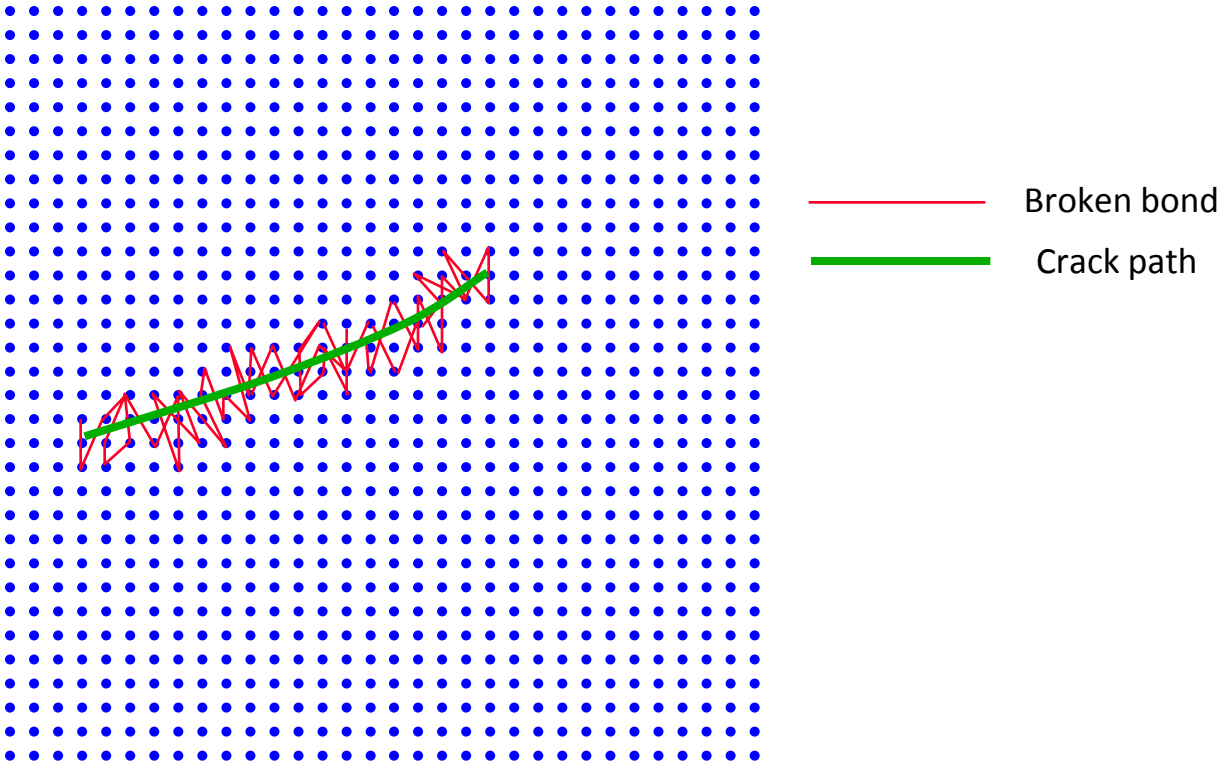
Damage due to bond breakage

- Recall: each bond carries a force.
- Damage is implemented at the bond level.
  - Bonds break irreversibly according to some criterion.
  - Broken bonds carry no force.
- Examples of criteria:
  - Critical bond strain (brittle).
  - Hashin failure criterion (composites).
  - Gurson (ductile metals).



Critical bond strain damage model

Autonomous crack growth

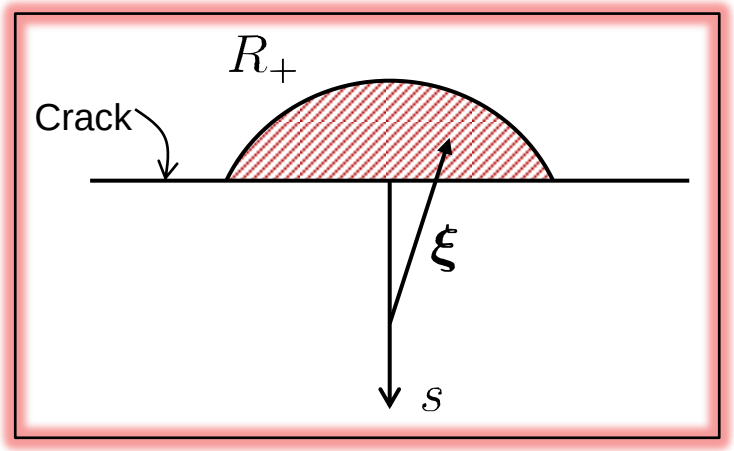


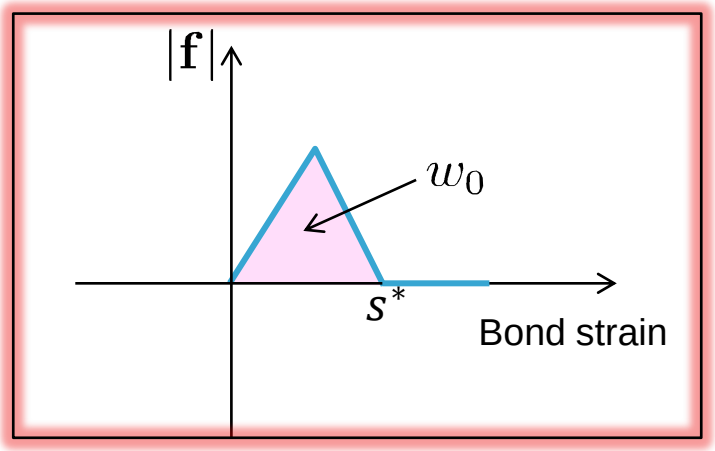
- When a bond breaks, its load is shifted to its neighbors, leading to progressive failure.

Critical bond strain: Relation to critical energy release rate

If the work required to break the bond $\boldsymbol{\xi}$ is $w_0(\boldsymbol{\xi})$, then the energy release rate is found by summing this work per unit crack area (J. Foster):

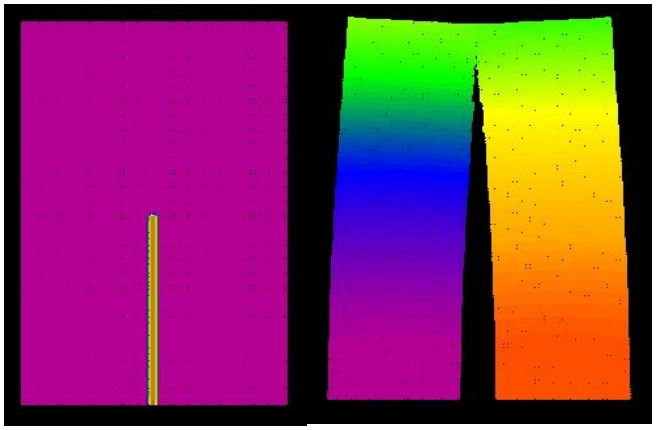
$$G = \int_0^{\delta} \int_{R_+} w_0(\boldsymbol{\xi})\, dV_{\boldsymbol{\xi}}\, ds$$

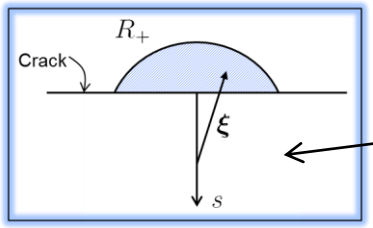




- Can then get the critical strain for bond breakage s^* in terms of G.
- Could also use the peridynamic J-integral as a bond breakage criterion.

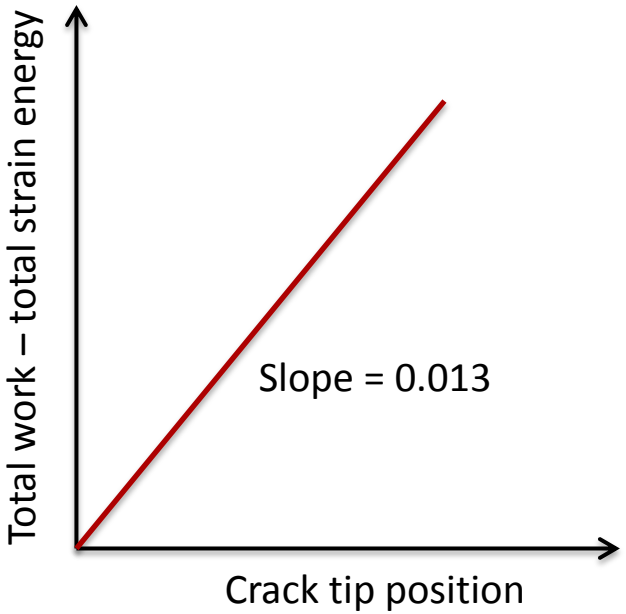
Constant bond failure strain reproduces the Griffith crack growth criterion





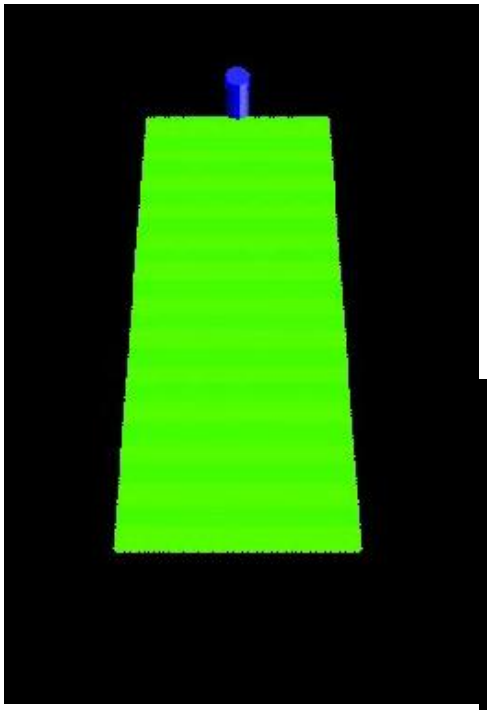
From bond properties, energy release rate should be

G = 0.013

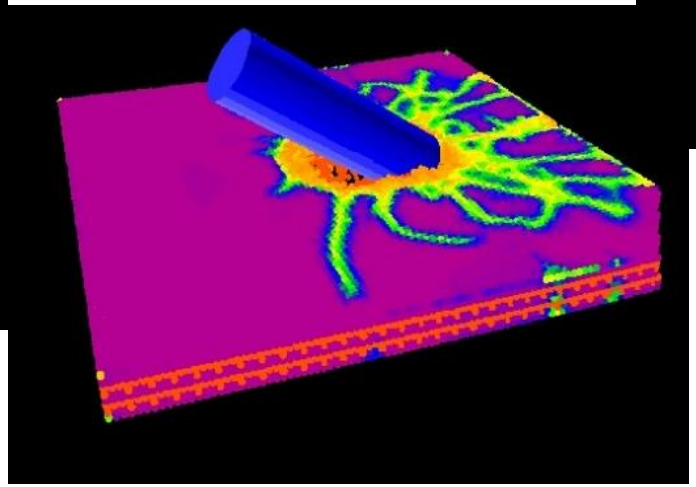


- This confirms that the energy consumed per unit crack growth area equals the expected value from bond breakage properties.

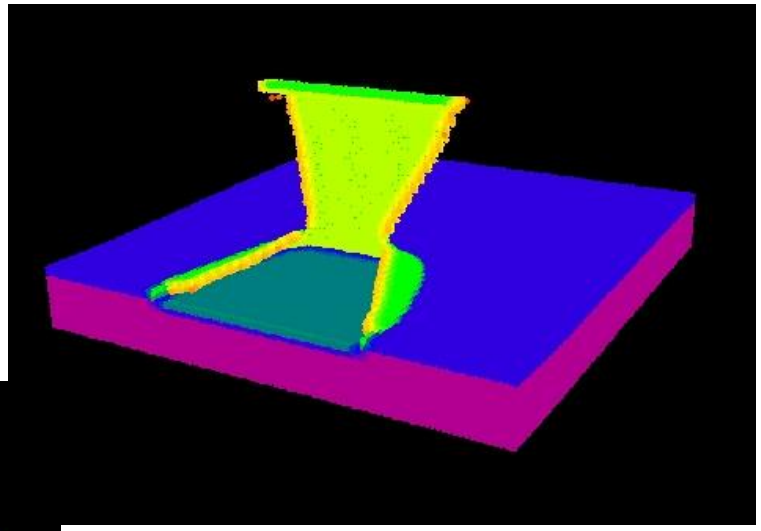
Treating discontinuities on an equal basis allows some subtleties in fracture to appear



Wavy crack path in a sheet (VIDEO)



Complex crack paths in reinforced concrete



Peeling and tearing of an adhesive membrane

Cyclic strain in a bond

- For a given bond $\boldsymbol{\xi}$, the *bond elongation* is the change in bond length:

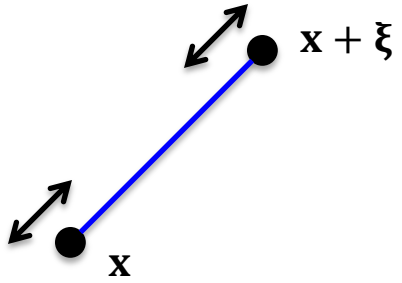
$$e = |\underline{\mathbf{Y}}\langle\boldsymbol{\xi}\rangle| - |\boldsymbol{\xi}| = |\mathbf{y}(\mathbf{x}+\boldsymbol{\xi}) - \mathbf{y}(\mathbf{x})|.$$

- The *bond strain* is the change in length over initial length:

$$s = \frac{e}{|\boldsymbol{\xi}|}.$$

- Let s^+ and s^- be the two extremes under cyclic loading of $\boldsymbol{\xi}$.
- The *cyclic bond strain* is defined by

$$\varepsilon = |s^+ - s^-|.$$

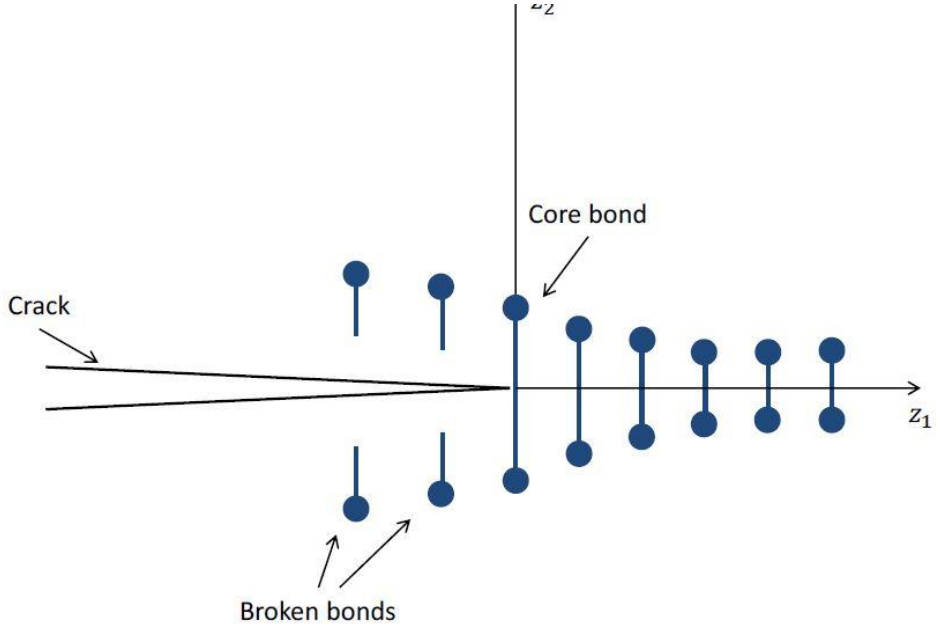


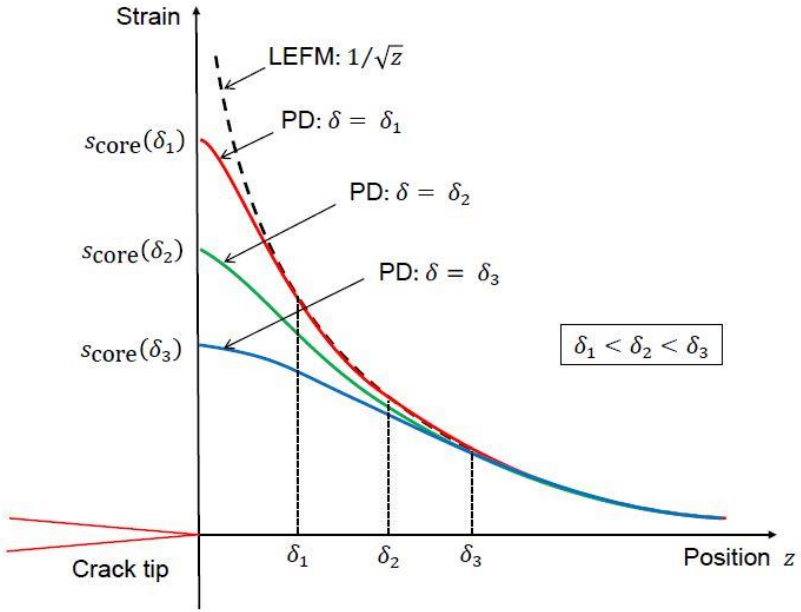
Structure of a crack tip field

- Let $\varepsilon_{\text{core}}(\delta)$ be the largest cyclic strain in any bond.
- Can show by a dimensional argument $\exists\ \hat{\varepsilon}_{\text{core}}$ such that

$$\varepsilon_{\text{core}}(\delta) = \hat{\varepsilon}_{\text{core}} \frac{\Delta K}{E\sqrt{\delta}}$$

where ΔK = cyclic stress intensity factor and E = modulus.





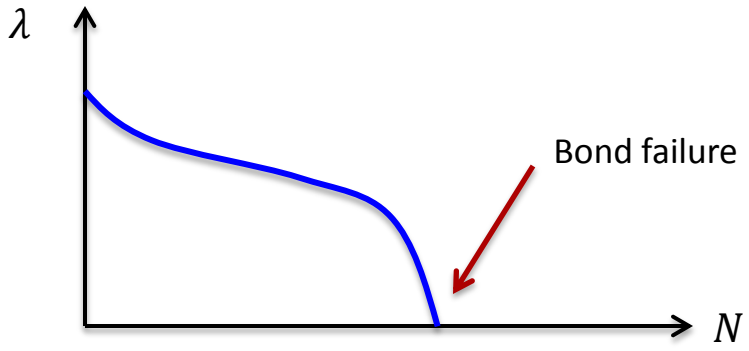
Remaining life of a bond

- Each bond in the body has a *remaining life* $\lambda(N)$ where N is the cycle number.
- The remaining life is monotonically decreasing over time.

$$\lambda(0) = 1, \qquad \dot{\lambda} \leq 0.$$

- The bond fails at the first cycle N when

$$\lambda(N) \leq 0.$$



Fatigue model

- The fatigue model specifies how the remaining life of each bond depends on the loading.

$$\frac{d\lambda}{dN}(N) = -A\varepsilon^m$$

  where A and m are constants and ε is the cyclic bond strain.

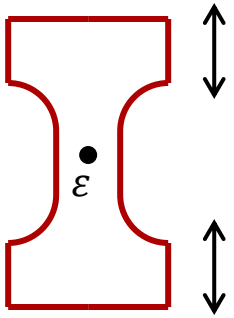
- The constants are calibrated separately for phases I and II (nucleation and growth).

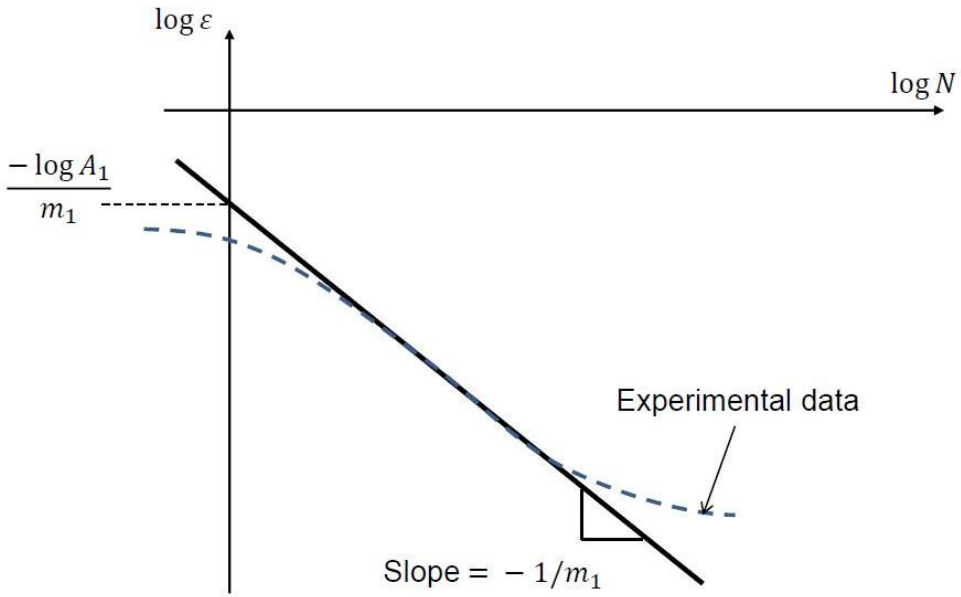
Phase I calibration from S-N data

- Run many cyclic loading tests at different values of ε (constant for each test).
- For each test, compute when damage starts:

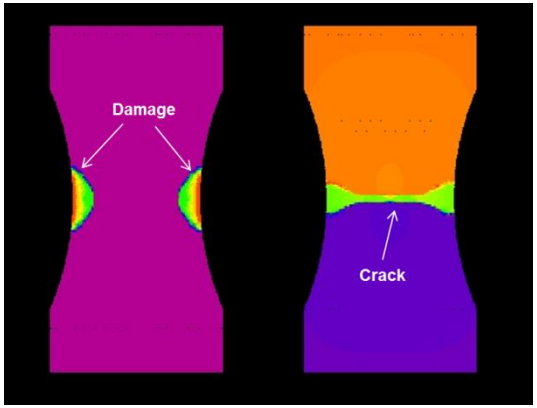
$$\frac{d\lambda}{dN}(N) = -A\varepsilon^m \qquad \Longrightarrow \qquad N = \frac{1}{A\varepsilon^m}.$$

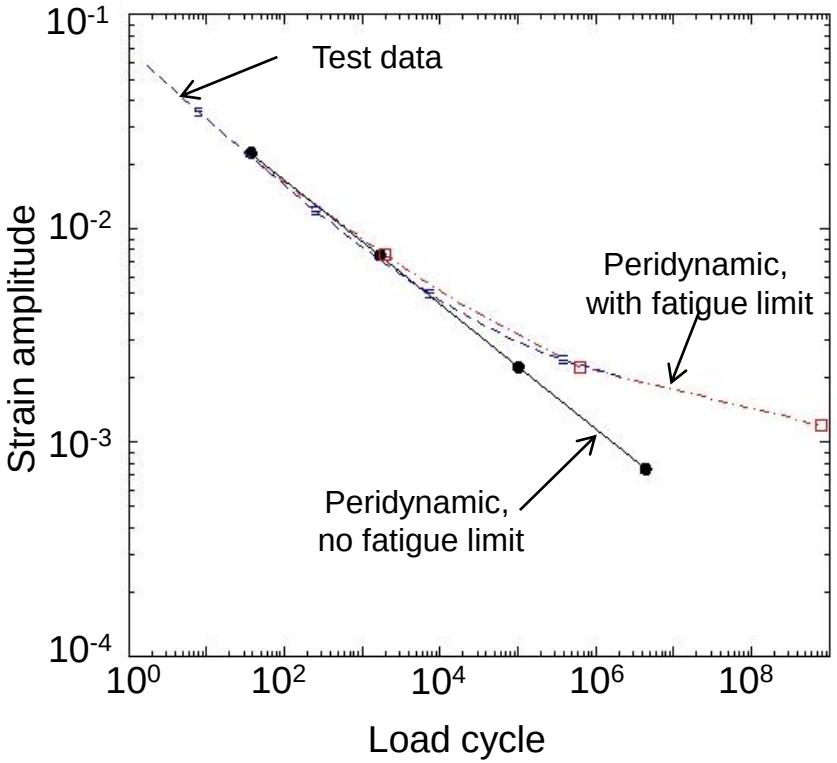
- Compare this to data on an ε-N plot, fit A and m.





Fatigue nucleation in aluminum alloy



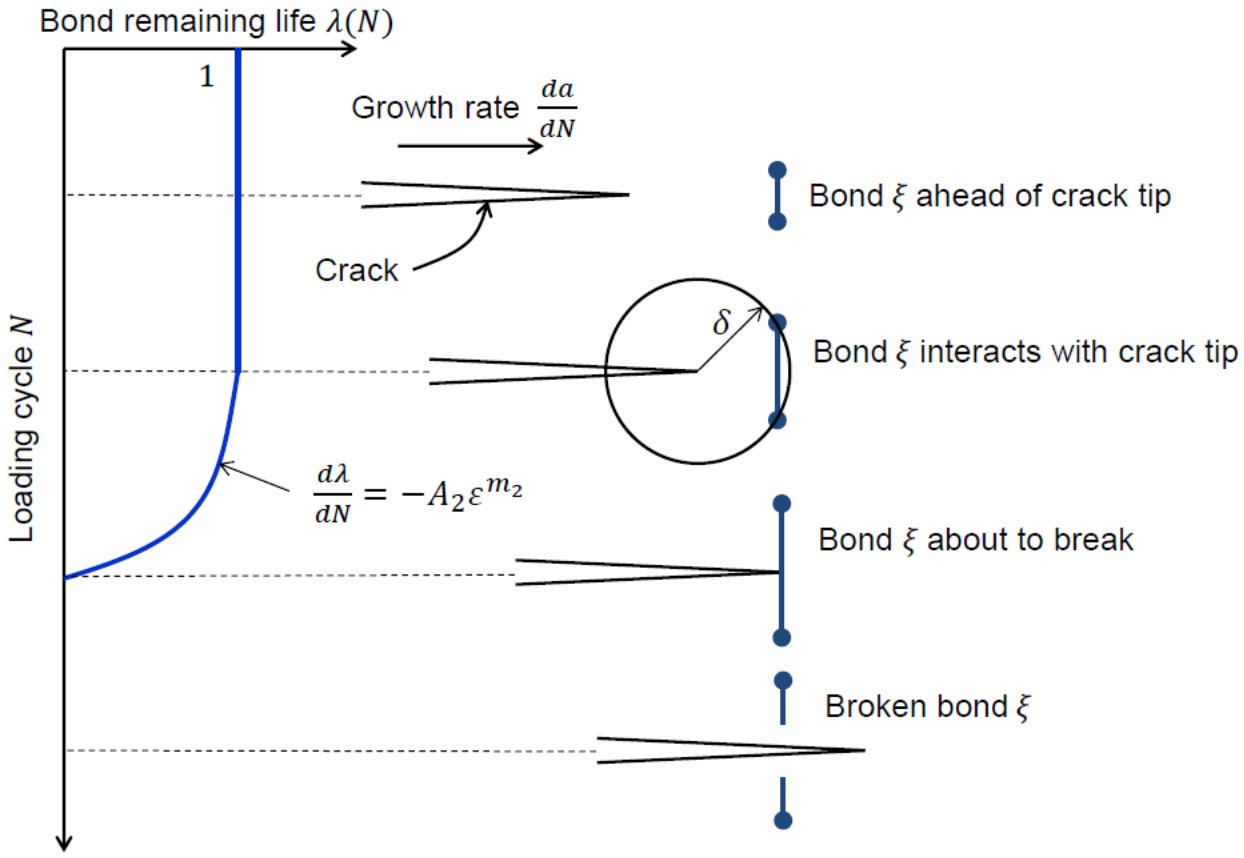


- Model with a fatigue limit:

$$\frac{d\lambda}{dN}(N) = -A\Big(\max(0, \varepsilon - \varepsilon_\infty)\Big)^m$$

Test data: T. Zhao and Y. Jiang. Fatigue of 7075-T651 aluminum alloy. International Journal of Fatigue, 30 (2008)834-849.

Growth: Bonds interact with the strain field near an approaching crack



Relate crack growth to remaining life

- Evolution of remaining life:

$$\lambda(\delta) - \lambda(0) = \int_0^\delta \frac{d\lambda}{dz}\, dz = \int_0^\delta \frac{d\lambda}{dN}\frac{dN}{dz}\, dz.$$

- Recall

$$\frac{d\lambda}{dN} = -A\varepsilon^m.$$

- Denote by da/dN the crack growth rate.

$$1 - 0 = \frac{A}{da/dN}\int_0^\delta \varepsilon^m(z)\, dz$$

- Cyclic strain ahead of a crack:

$$\varepsilon(z) = \varepsilon_{\text{core}} f\left(\frac{z}{\delta}\right) = \frac{\Delta K}{E\sqrt{\delta}} f\left(\frac{z}{\delta}\right).$$

- Thus, for some c,

$$\frac{da}{dN} = cA\Delta K^m$$

Phase II calibration from Paris Law data

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- Now have

$$\frac{da}{dN} = cA\Delta K^m$$

where c and m are as yet unknown.

- Assume the Paris Law holds:

$$\frac{da}{dN} = C\Delta K^M$$

where C and M are constants that can be found from test data.

- Conclude

$$m = M.$$

- Need to do one computational simulation with an assumed value $C = 1$ to evaluate A.

Summary so far

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- Each bond has a remaining life $\lambda(N)$:

$$\lambda(0) = 1, \qquad \frac{d\lambda}{dN}(N) = -A\varepsilon^m, \qquad \lambda \leq 0 \text{ means failure.}$$

- In Phase I, use A and m from S-N data.
- In Phase II, use a different calibration from Paris law data.

Time mapping permits very large N

- We can avoid modeling each cycle explicitly.
- Define the *loading ratio* by

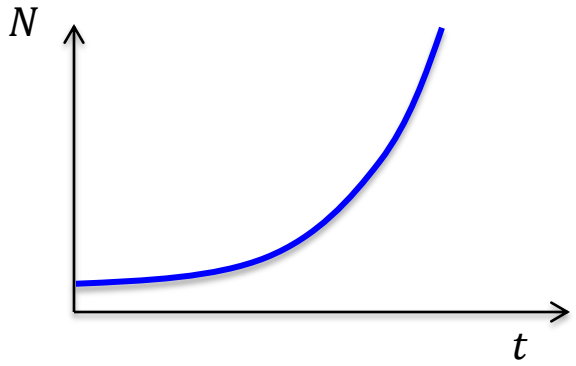
$$R = \frac{s^-}{s^+} \qquad \Longrightarrow \qquad \varepsilon = |s^+ - s^-| = |(1-R)s^+|.$$

- Map t to N:

$$N = e^{t/\tau}$$

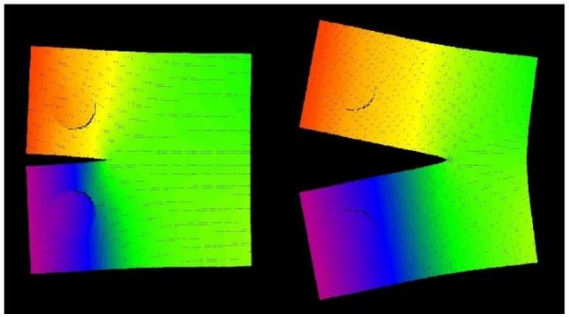
  where τ is a constant chosen according to convenience.

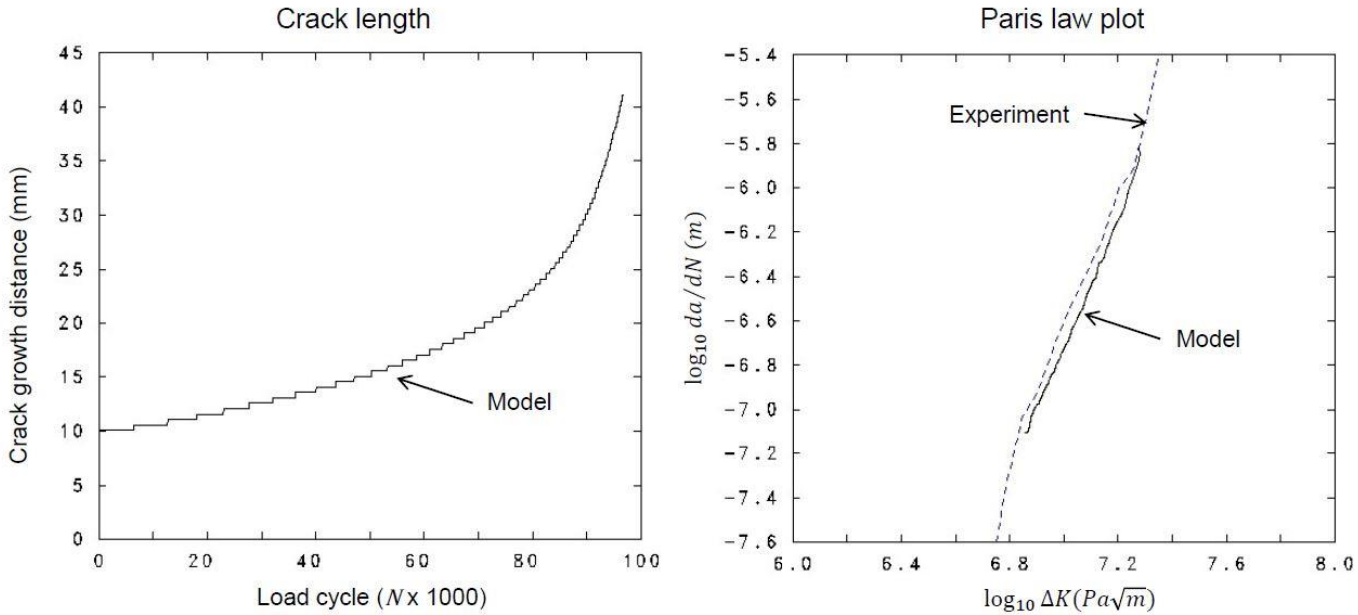
- Fatigue model in terms of t instead of N:



$$\frac{d\lambda}{dt} = \frac{d\lambda}{dN}\frac{dN}{dt} = \frac{-|1-R|AN}{\tau}|s^+|^m.$$

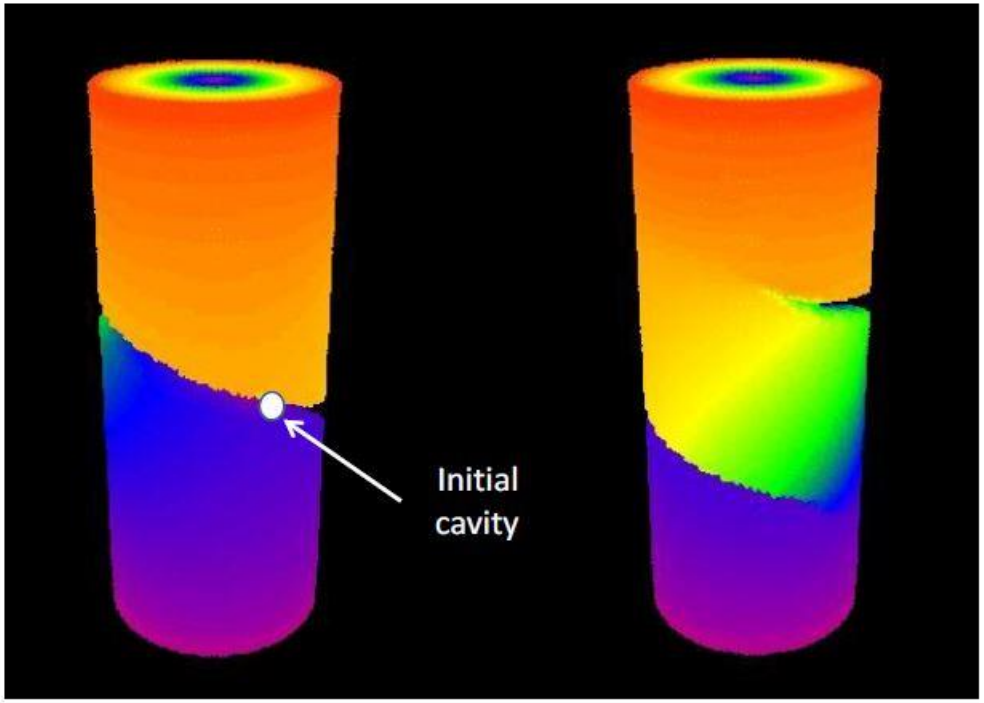
Fatigue crack growth in aluminum





Test data: T. Zhao, J. Zhang, and Y. Jiang. A study of fatigue crack growth of 7075-T651 aluminum alloy. International Journal of Fatigue, 30 (2008) 1169-1180.

Spiral crack in a rod under torsion



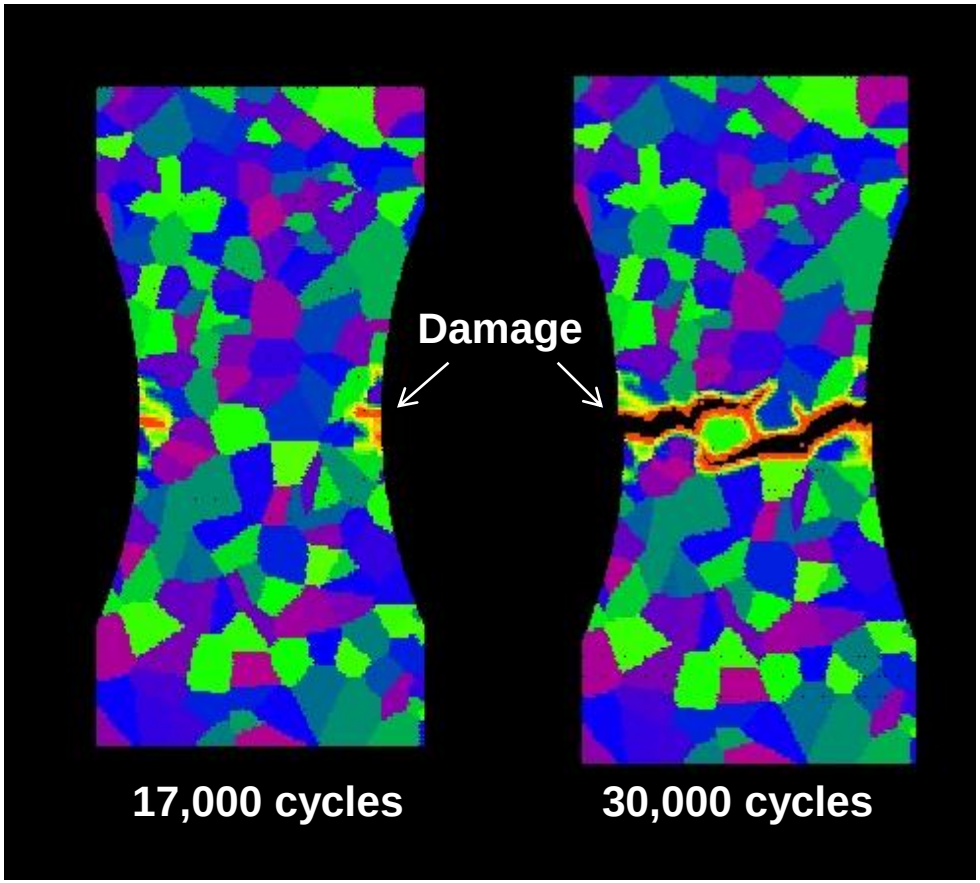
Mesoscale:
Fatigue cracks at grain boundaries

- Recall the peridynamic fatigue model: For a given bond,

$$\lambda(0) = 1, \qquad \frac{d\lambda}{dN} = -A\varepsilon^m$$

- Set:

$A = 5$ for bonds within a grain
$A = 50$ for bonds between grains



Fatigue crack growth between grains represented as Voronoi cells

Corrosion fatigue: Diffusion coupled with damage

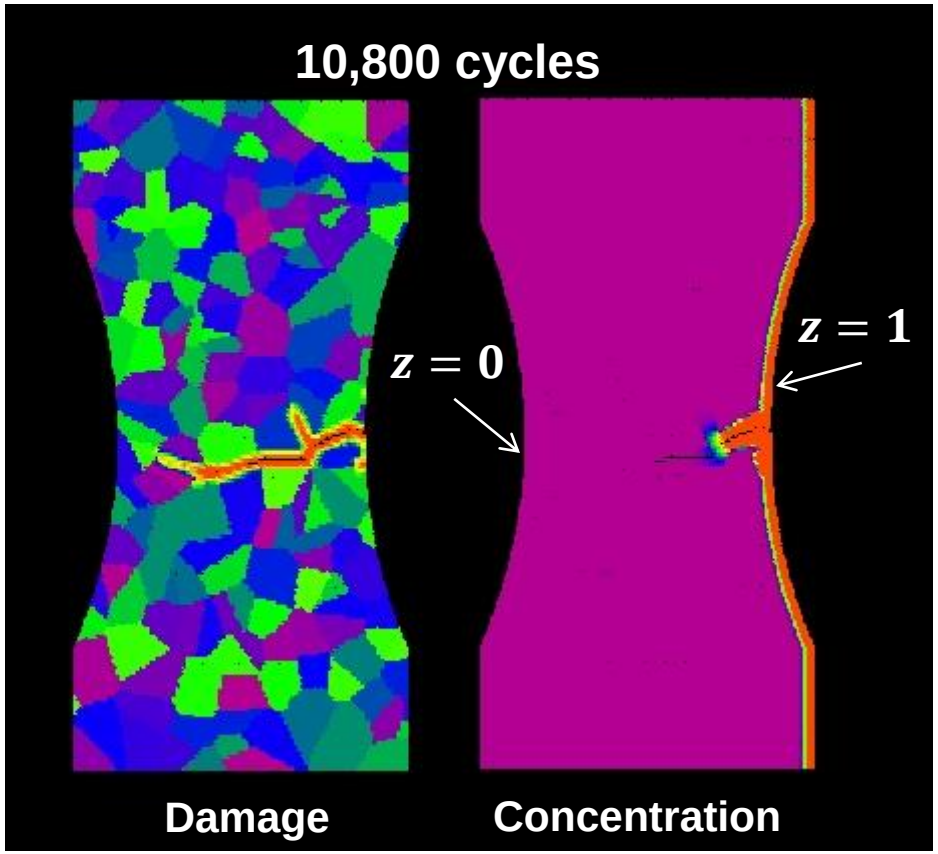
- Let z =concentration of a contaminant.
- Bond life loss rate depends on concentration:

$$\lambda(0) = 1, \qquad \frac{d\lambda}{dN} = -A(z)\varepsilon^m$$

- Peridynamic diffusion model:

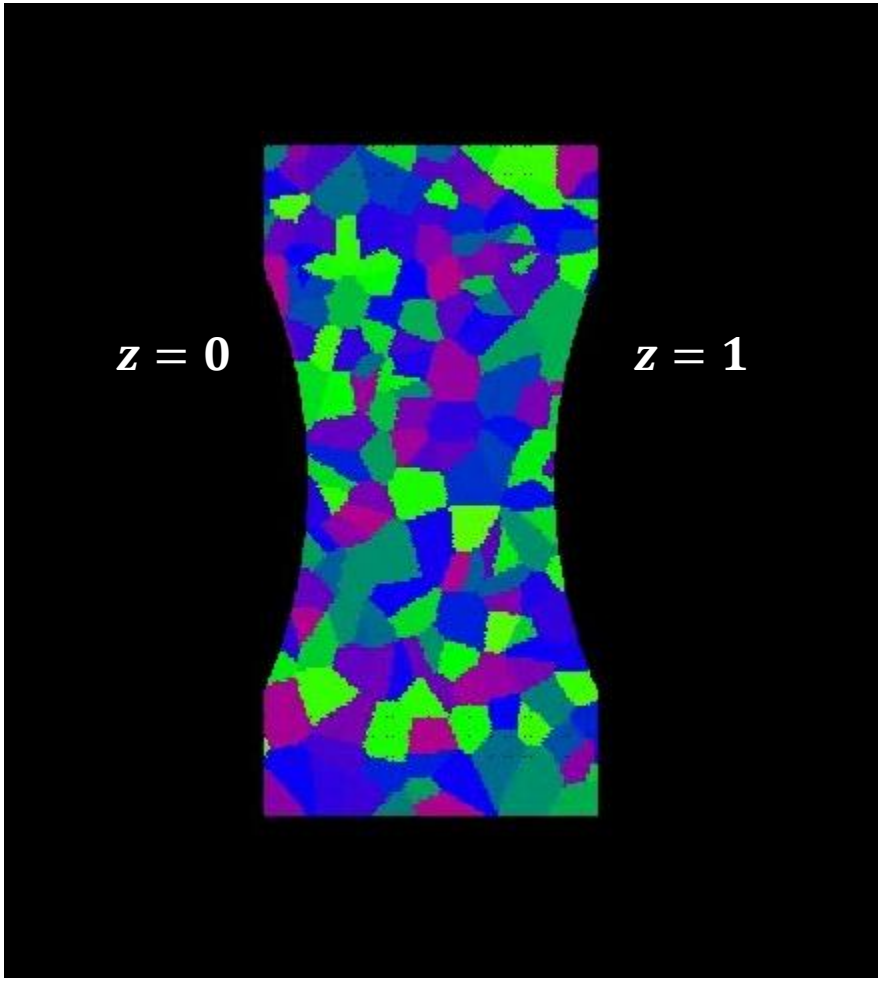
$$\dot{z}(x,t) = \int C(q,x)\big(z(q,t) - z(x,t)\big)dV_q$$

where $C(q,x)$is much larger for damaged bonds than undamaged.



Example: nucleation occurs at
N =2100 with corrosion
N =12,000 without corrosion

Corrosion fatigue:
Diffusion coupled with damage



Discussion

- Method retains the main advantages of peridynamics.
  - Autonomous crack growth
  - Includes both nucleation and growth phases
  - Permits interaction between multiple cracks
  - Arbitrary crack path in 3D.
- A simple enhancement allows a spectrum of loading frequencies
  - This is a peridynamic version of Miner's rule.

Extra slides

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Peridynamics basics: States

- A *peridynamic state* is a mapping on bonds in a family.
- We write:

$$\mathbf{u} = \underline{\mathbf{A}}\langle \boldsymbol{\xi} \rangle$$

  where $\boldsymbol{\xi}$ is a bond, $\underline{\mathbf{A}}$ is a state, and $\mathbf{u}$ is some vector.

- States play a role in peridynamics similar to that of second order tensors in the local theory.

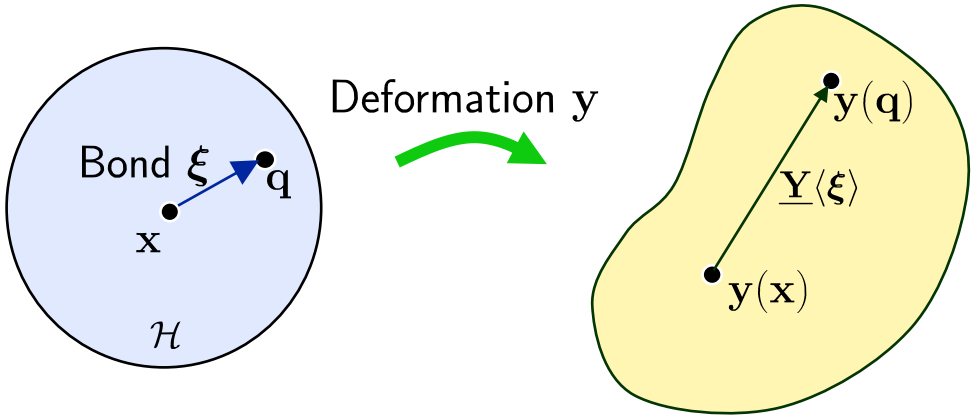
Peridynamics basics: Kinematics

- The *deformation state* is the function that maps each bond $\boldsymbol{\xi}$ into its deformed image:

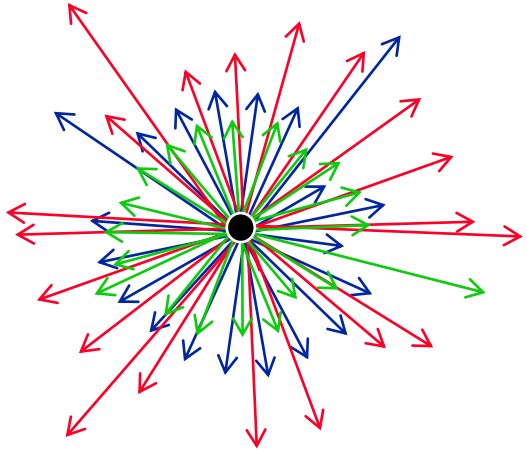
$$\underline{\mathbf{Y}}\langle\boldsymbol{\xi}\rangle = \mathbf{y}(\mathbf{q}) - \mathbf{y}(\mathbf{x})$$

where $\mathbf{y}$ is the deformation and

$$\boldsymbol{\xi} = \mathbf{q} - \mathbf{x}.$$



Undeformed family of $\mathbf{x}$ Deformed family of $\mathbf{x}$



Deformed images of bonds:
State description allows complexity

Peridynamics basics: Bonds and bond force density

- The vector from $\mathbf{x}$ to any point $\mathbf{q}$ in its family in the reference configuration is called a *bond*.

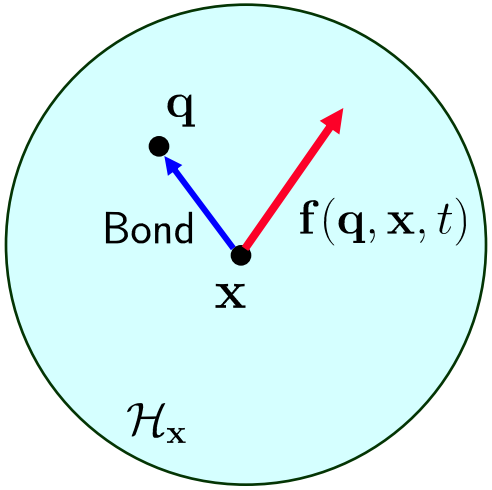
$$\boldsymbol{\xi} = \mathbf{q} - \mathbf{x}$$

- Each bond has a *pairwise force density* vector that is applied at both points:

$$\mathbf{f}(\mathbf{q}, \mathbf{x}, t).$$

- Equation of motion is an integro-differential equation, not a PDE:

$$\rho(\mathbf{x})\ddot{\mathbf{y}}(\mathbf{x}, t) = \int_{\mathcal{H}} \mathbf{f}(\mathbf{q}, \mathbf{x}, t)\, dV_{\mathbf{q}} + \mathbf{b}(\mathbf{x}, t).$$



Peridynamics basics: Elastic materials

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- A peridynamic elastic material has strain energy density given by

$$W(\underline{\mathbf{Y}}).$$

- The force state is given by

$$\underline{\hat{\mathbf{T}}}(\underline{\mathbf{Y}}) = W_{\underline{\mathbf{Y}}}(\underline{\mathbf{Y}})$$

  where $W_{\underline{\mathbf{Y}}}$ is the Frechet derivative of the strain energy density.

Some results about peridynamics

- For any choice of horizon, we can fit material model parameters to match the bulk properties and energy release rate.
  - Using nonlocality, can obtain material model parameters from wave dispersion curves (Weckner).
- Coupled coarse scale and fine scale evolution equations can be derived for composites (Lipton and Alali).
- A set of discrete particles interacting through any multibody potential can be represented exactly as a peridynamic body.
- Well posedness has been established under certain conditions (Mangesha, Du, Gunzburger, Lehoucq).